

# Analyzing the Sepsic Converter

*In the last issue, we talked about the simplest of all converters, the buck converter, and showed how its control transfer functions could be extraordinarily complex. In this issue, we'll go to the other end of the spectrum, and look at a converter that is far more complex, yet is often used by engineers who are unaware of the difficulties that follow.*

*By Dr. Ray Ridley, Ridley Engineering*

## The Sepsic Converter

The most basic converter that we looked at last month is the buck converter. It is so named because it always steps down, or bucks, the input voltage. The output of the converter is given by:

$$V_o = DV_g$$

Interchange the input and the output of the buck converter, and you get the second basic converter – the boost. The boost always steps up, hence its name. The output voltage is always higher than the input voltage, and is given by:

$$V_o = \frac{1}{D'}V_g$$

What if you have an application where you need to both step up and step down, depending on the input and output voltage? You could use two cascaded converters – a buck and a boost. Unfortunately, this requires two separate controllers and switches. It is, however, a good solution in many cases.

The buck-boost converter has the desired step up and step down functions:

$$V_o = \frac{-D}{D'}V_g$$

The output is inverted. A flyback converter (isolated buck-boost) requires a transformer instead of just an inductor, adding to the complexity of the development.

One converter that provides the needed input-to-output gain is the Sepsic (single-ended primary inductor converter) converter. A Sepsic converter is shown in Fig. 1. It has become popular in recent



years in battery-powered systems that must step up or down depending upon the charge level of the battery.

Fig. 2 shows the circuit when the power switch is turned on. The first inductor, L1, is charged from the input voltage source during this time. The second inductor takes energy from the first capacitor, and the output capacitor is left to provide the load current. The fact that both L1 and L2 are disconnected from the load when the switch is on leads to complex control characteristics, as we will see later.

When the power switch is turned off, the first inductor charges the capacitor C1 and also provides current to the load, as shown in Fig. 3. The second inductor is also connected to the load during this time.

The output capacitor sees a pulse of current during the off time, making it inherently noisier than a buck converter.

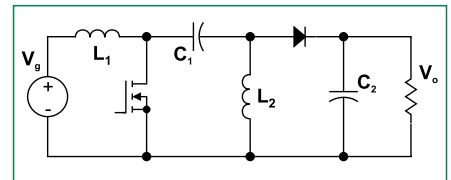


Figure 1. The Sepsic converter can both step up and step down the input voltage, while maintaining the same polarity and the same ground reference for the input and output.

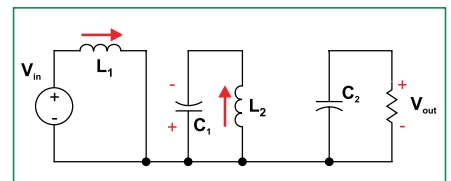


Figure 2. When the switch is turned on, the input inductor is charged from the source, and the second inductor is charged from the first capacitor. No energy is supplied to the load capacitor during this time. Inductor current and capacitor voltage polarities are marked in this figure.

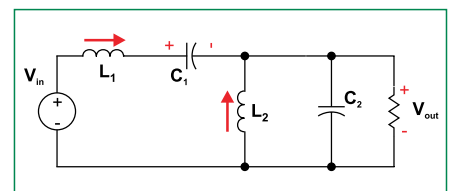


Figure 3. With the switch off, both inductors provide current to the load capacitor.

The input current is non-pulsating, a distinct advantage in running from a battery supply.

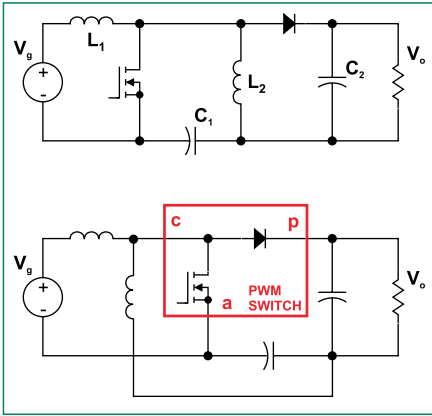


Figure 4. In order to take advantage of Vorpérian’s PWM switch model, the circuit elements must first be rearranged. The function of the original topology is retained when the capacitor is moved, and the second inductor is redrawn.

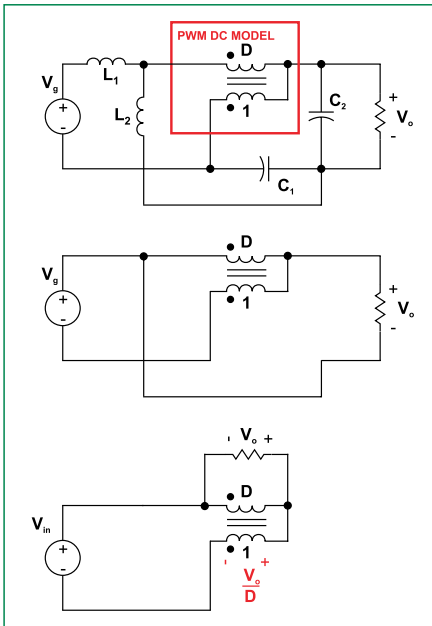


Figure 5. For DC analysis, the small signal sources are set to zero, inductors become short circuits, and capacitors become open circuits. After the circuit is redrawn, it is a trivial matter to write KVL around the outer loop of the circuit to solve for the conversion gain of the converter.

**The PWM Switch Model in the Sepic Converter**

The best way to analyze both the AC and DC characteristics of the Sepic converter is by using the PWM switch model, developed by Dr. Vatché Vorpérian in 1986. Some minor circuit manipulations are first needed to reveal the location of the switch model, and this is

shown in Fig. 4.

First, capacitor C1 is moved to the bottom branch of the converter. Then, inductor L2 is pulled over to the left, keeping its ends connected to the same nodes of the circuit. This reveals the PWM switch model of the converter, with its active, passive, and common ports, allowing us to use well-established analysis results for this converter.

For more background on the PWM switch model, the text book “Fast Analytical Techniques for Electrical and Electronic Circuits” [1] is highly recommended.

**DC Analysis of the Sepic Converter**

Fig. 5 shows the equivalent circuit of the Sepic converter with the DC portion of the PWM switch model in place. The DC model is just a 1:D transformer. We replace the inductors with short circuits, and the capacitors with open circuits for the DC analysis. You can, if you like, include any parasitic resistances in the model [2], but that’s beyond the scope of this article.

After the circuit is manipulated as shown in the figure, we can write the KVL equation around the outer loop of the converter:

$$V_g + V_o - \frac{1}{D} V_o = 0$$

Rearranging gives:

$$V_g = \left( \frac{1}{D} - 1 \right) V_o = \frac{D'}{D} V_o$$

And the DC gain is given by:

$$V_o = \frac{D}{D'} V_g$$

Here we see the ability of the converter to step up or down, with a gain of 1 when D=0.5. Unlike the buck-boost and Cuk converters, the output is not inverted.

**AC Analysis of the Sepic Converter**

You won’t find a complete analysis of the Sepic converter anywhere in printed literature. What you will find are application notes with comments like, “the Sepic is not well-understood.” Despite the lack of documentation for the converter, engineers continue to use it when applicable.

Proper small-signal analysis of the Sepic converter is a difficult analytical task, only made practical by advanced circuit analysis techniques originally developed by Dr. David Middlebrook and continued by Vorpérian. [1]

If you’re going to build a Sepic, as a minimum, you need to understand the control characteristics. Fortunately, Vorpérian’s work is now available for this converter, and you can download the complete analysis notes. [2]

The simplified analysis of the Sepic converter, derived in detail in [2], ignores parasitic resistances of the inductors and capacitors, and yields the following result for the control-to-output transfer function:

$$\frac{v_o(s)}{d(s)} \approx \frac{1}{D^2} \frac{\left( 1 - s \frac{L_1 D^2}{R} \right) \left( 1 - s \frac{C_1 (L_1 + L_2) R D^2}{L_1} + s^2 \frac{L_2 C_1}{D} \right)}{\left( 1 + \frac{s}{\omega_{o1} Q_1} + \frac{s^2}{\omega_{o1}^2} \right) \left( 1 + \frac{s}{\omega_{o2} Q_2} + \frac{s^2}{\omega_{o2}^2} \right)}$$

Where

$$\omega_{o1} \approx \frac{1}{\sqrt{L_1 \left( C_2 \frac{D^2}{D^2} + C_1 \right) + L_2 (C_1 + C_2)}}$$

$$Q_1 \approx \frac{R}{\omega_{o1} \left( L_1 \frac{D^2}{D^2} + L_2 \right)}$$

$$\omega_{o2} \approx \sqrt{\frac{1}{L_2 \frac{C_1}{D^2} \parallel \frac{C_2}{D^2}} + \frac{1}{L_1 C_1 \parallel C_2}}$$

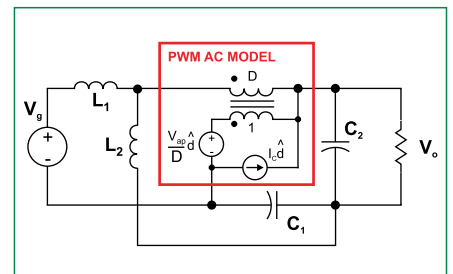


Figure 6. The small-signal AC sources are included in the switch model, and we can either solve the analysis by hand, or use PSpice to plot desired transfer functions. The hand analysis is crucial for symbolic expressions and design equations.

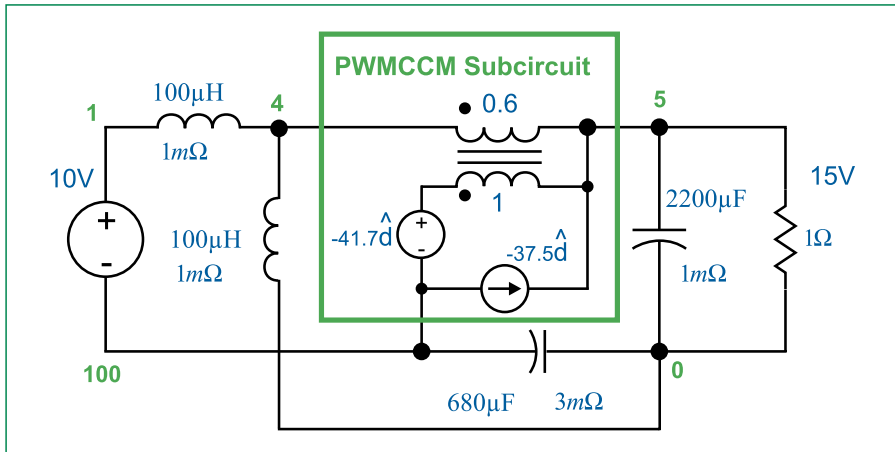


Figure 7. Analysis can also be done with PSpice. This figure shows a specific design example for a 15 W converter. Parasitic resistances are included in the PSpice model.

$$Q_2 \approx \frac{R}{\omega_{o2} (L_1 + L_2) \frac{C_1}{C_2} \frac{\omega_{o1}^2}{\omega_{o2}^2}}$$

As you can see from these expressions, the “simplified” analysis is anything but simple. Including the parasitic resistances greatly complicates the analysis, but may be necessary for worst-case analysis of the Sepic converter. The analysis of this converter involves the use of the powerful extra element theorem, and Vorpérian’s book on circuit analysis techniques.<sup>[1]</sup>

In addition to the inevitable fourth-order denominator of the Sepic, the most important features to note in the control transfer function are the terms in the numerator. The first term is a single right-half-plane (RHP) zero. Right-half-plane zeros are a result of converters where the response to an increased duty cycle is to initially decrease the output voltage.

When the power switch is turned on, the first inductor is disconnected from the load, and this directly gives rise to the first-order RHP zero. Notice that the expression only depends on the input inductor, L<sub>1</sub>, the load resistor, R, and the duty cycle.

The complex RHP zeros arise from the fact that turning on the switch disconnects the second inductor from the load. These zeros will actually move with the values of parasitic resistors in the circuit, so careful analysis of your converter is needed to ensure stability under all conditions.

### PSpice Modeling of the Sepic Converter

The analytical solution above does not include all of the parasitic circuit elements. As you will see from <sup>[2]</sup>, there is a prodigious amount of work to be done even without the resistances.

We can also use PSpice to help understand the Sepic better. Fig. 7 shows the circuit model for a specific numerical application of the Sepic, and it includes resistances which will affect the stability of the converter, sometimes in dramatic ways.

The PSpice file listing can be downloaded from <sup>[2]</sup> so you can reproduce these results to analyze your own Sepic converter.

Fig. 8 shows the result of the PSpice analysis. The two resonant frequencies predicted by the hand analysis can clearly be seen in the transfer function plot. What is remarkable is the extreme amount of phase shift after the second resonance. This is caused by the delay of the second pair of poles, and the additional delay of the complex RHP zeros. The total phase delay through the converter is an astonishing 630 degrees. Controlling this converter at a frequency beyond the second resonance is impossible.

### Summary

The Sepic converter definitely has some select applications where it is the topology of choice. How do designers get away with building such a convert-

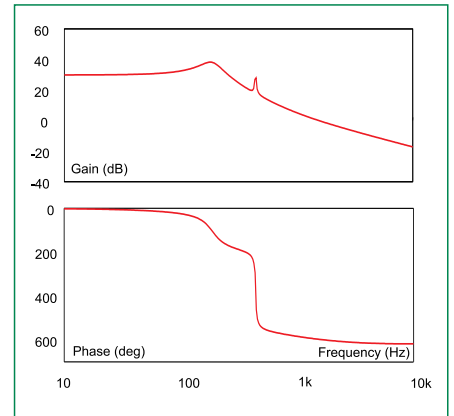


Figure 8. This shows the control-to-output transfer function for the Sepic converter. With low values of damping resistors, the converter has four poles, and three right-half-plane zeros. This results in an extreme phase delay of 630 degrees!

er? There are several possibilities. First, the dynamic and step load requirements on the system may be very benign, with no reason to design a loop with high bandwidth. This allows the loop gain to be reduced below 0 dB before the extreme phase delay of the second resonance.

Secondly, in many practical cases, the parasitic resistances of the circuit move the RHP zeros to the left half plane, greatly reducing the phase delay. This can also be done with the addition of damping networks to the power stage, a topic beyond the scope of this article.

Thirdly, some engineers do not build a proper Sepic. In some application notes, the two inductors are wound on a single toroidal core, which provides almost unity coupling between the two. In this case, the circuit no longer works as a proper Sepic. Don’t fall into this design trap - the circuit will be far from optimum.

### Additional Reading

[1] “Fast Analytical Techniques for Electrical and Electronic Circuits”, Vatché Vorpérian, Cambridge University Press 2002. ISBN 0 521 62442 8.

[2] <http://www.switchingpowermagazine.com>. Click on Articles and Sepic Analysis Notes.

# Will Switching Power Supply Design go the Way of Linear Regulator Design?

*Design Tips is devoted to the complex and intriguing issues that continue to make switching power supply design a challenging field of expertise. I hope to illustrate, through each design tip, the need for diligence in designing power supplies.*

*By Dr. Ray Ridley, Ridley Engineering*

Outside the power supply design community, there is a common misperception that power supply design is easy. Many companies minimize the time and attention given to their pre-production designs, without regard to the costly consequences. This has become obvious over the past few years with a flood of product recalls over power and heat issues.

There is no doubt that power supply design is a mature industry. Semiconductor companies are integrating standard functions into advanced chips with more capability inside the chip, and fewer parts on the board.

As the switching power supply functions become incorporated inside the chip, however, we lose design flexibility and access to crucial functions. I'll talk in a future column about the parts that are being integrated, and why I personally prefer access to many of them with discrete designs. For now, we'll focus on just one of the overlooked functions—the feedback control loop.

## Linear Regulators

Integration of discrete power circuits occurred once before—thirty years ago. This was an era before switchers, when the



industry was dominated by linear regulators. Sophisticated designs were generated by experienced engineers to optimize parameters such as the minimum dropout voltage, transient response time, thermal characteristics, and efficiency.

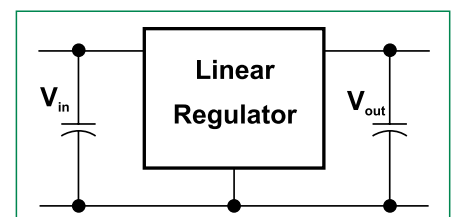
Designers were highly knowledgeable in transistor characteristics, thermal design, and feedback analysis. Since all regulators use error amplifiers to precisely set the output voltage, feedback analysis and measurement were part of the design procedure for an optimized system.

Later, standard solutions arrived in the industry, leading to integration of the linear regulator. Since then, few of us

build our own. In the process, access to the feedback loop has been lost, but no one seems to be concerned about this. Why not? The integration of the linear regulator went fairly smoothly, and there are three reasons for this: 1) predictability, 2) consistency with line and load variations, and 3) low noise.

Feedback for the linear regulator is quite straightforward. The small-signal model is a current source feeding a capacitor and load resistor. The only variation in the design of a control loop for the system is in the impedance of the output capacitor. Apart from this, the system is predictable, and can easily be simulated and modeled. Modeling results typically agree closely with measurements.

Once the linear regulator design is



*Figure 1: Once designed with discrete components, the three-terminal linear regulator is now almost always fully integrated. There is no longer any access to the control loop.*

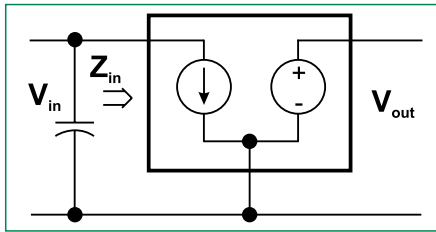


Figure 2: As long as the linear regulator control loop is stable, the circuit model looks like a current source on the input (infinite impedance) and a voltage source on the output (very low impedance). Very little noise is introduced into the system.

placed in the system, it can be modeled as shown in Figure 2. The input impedance of the linear regulator is a current source. Regardless of changes in the input voltage, the current draw is fixed, equal to the output voltage divided by the load resistance. The output of the regulator model is just a voltage source.

Since the linear regulator does not generate any significant noise, we have no need to introduce any complex filtering on the system board. As a result, placement and integration of linear regulators on the board is a job that is almost trivial. A power electronics designer is not needed.

Does the future hold the same fate for the switching power supply? Will integration of the controller functions with power devices and auxiliary circuits render the switching power supply design a simple process?

Well, let's not rush to conclusions. As we'll see below, even the simplest switching power supply can have tremendous complexity.

**PWM Switching Regulators**

The most basic of all switching regulators is the buck converter, shown in Figure 3. It consists of just four power components, and a feedback loop, shown in red. It looks very simple, but as power designers have realized for decades, there is a great deal of hidden complexity in this system.

Once you are past the first step of turning the semiconductor devices on and off, a circuit remains that generates

large amounts of noise. In addition it needs a feedback loop designed around it for a stable, regulated output. This is not always an easy task, especially if you are dealing with synchronous rectifiers or isolated converters.

We'll talk about the turn on and turn off issues later. Meanwhile, let's look at the control loop issues. The first step in dealing with control of any system is to properly understand its plant characteristics. For the buck converter in Figure 3, we are interested in power stage behavior. We can measure a transfer function that shows this by looking at the ratio of  $V_o/V_i$  while injecting a swept

sinusoidal source into the system. This can be accomplished by either a simulation program, or on the hardware as described in reference [1].

The blue and red curves of Figure 4 show the result of this for a buck converter operating at high and low line input. It's basically just an LC filter characteristic with its gain dependent on the input voltage.

If the load is reduced sufficiently, the converter will enter discontinuous mode. This is shown in the green curve (this does not happen with synchronous

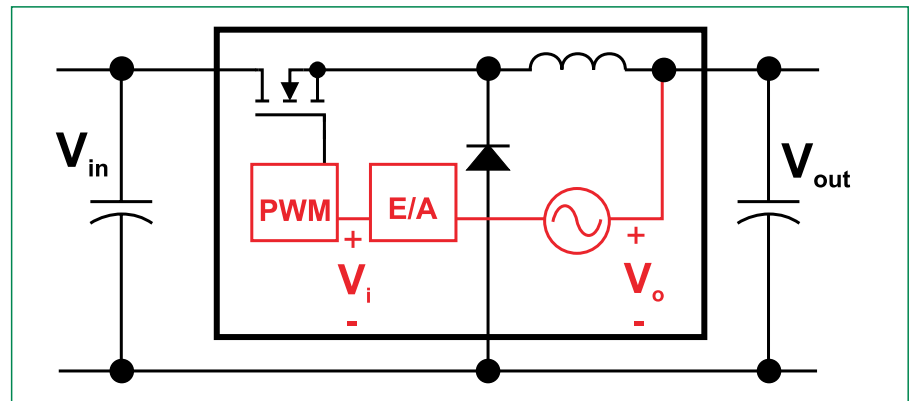


Figure 3: The apparently simple buck converter power stage consists of a switch, diode (or synchronous rectifier), inductor and capacitor. Feedback components are shown in red, with signal injection for transfer function measurements.

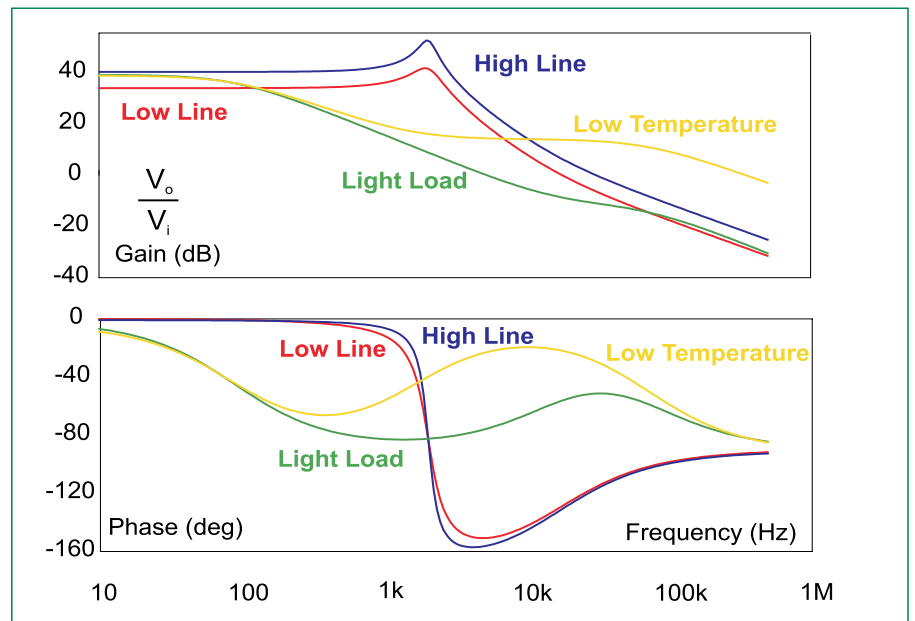


Figure 4: This bode plot shows how much the control characteristics can change with variations in input line, output load, and temperature. At a typical crossover frequency of 10 kHz, the phase of the converter can vary by as much as 120 degrees, presenting a formidable challenge to the designer.

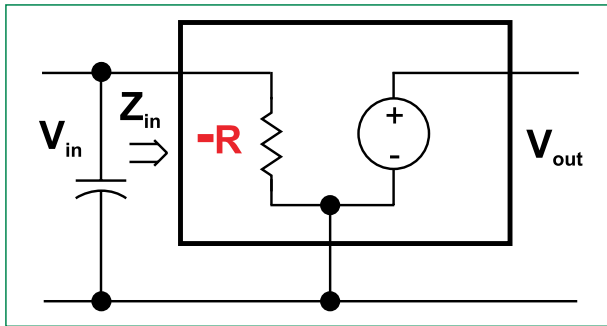


Figure 5: Even if you manage to stabilize the buck converter by itself, it can become unstable again when you apply proper filters to eliminate noise. The negative input resistance can destabilize the passive components on the rest of your circuit board.

rectifiers). There's a big change in the characteristics. Further change can occur if you have electrolytic capacitors in your system, and the temperature is reduced below freezing.

The resulting range of gain and phase curves with all these variations is much worse than a linear regulator loop would ever see. This includes only the basic effects that can occur with

switching regulators. Many more variations occur with multiple outputs and noise issues<sup>[3]</sup>.

Now let's take a look at what happens if you succeed in building an acceptable control loop with these variable characteristics. A simplified model is shown in Figure 5. The input impedance of the power supply looks like a negative resistor,

which is quite different from the current source of the linear regulator. A negative resistor is a troublesome component in a system, frequently leading to oscillation. With input filters, it can actively excite the LC filters needed for the power supply under certain conditions<sup>[2]</sup>, so great care must be paid to the choice of filter components.

The effect of the filter can be seen in

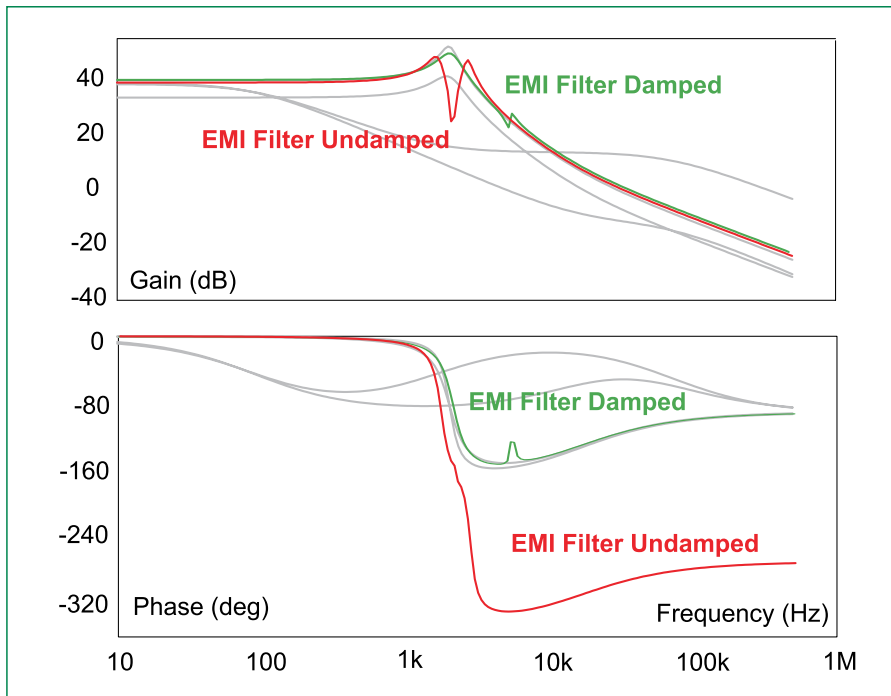


Figure 6: Switching power supplies are noisy. LC filters must be to attenuate the noise. The green curve in this figure shows the effect on control characteristics with a reasonably well-designed filter. The red curve shows what happens when the power supply is unstable. Notice the additional 180 degrees phase delay where the input filter creates a pair of complex RHP zeros in the control loop. The total variation in phase at 10 kHz for the complete range of the power supply is now over 300 degrees!

Figure 6. In the green curve, there is a small blip in the gain and phase of the power supply. The design is still OK, but it's interacting with the filter. In the red curve, there is severe interaction, and the power stage suddenly has an additional 180 degrees phase delay in the control characteristic. This power supply will most likely be an unstable design.

There are many more situations that can be created, but hopefully the point has been made. Switching power supplies will never be drop-in designs, like linear regulators. Proper engineering is critical to incorporate the switcher reliably into the system.

The design of a switcher is still a complex task that needs proper engineering attention. Make sure that you budget time and money accordingly, and bring the proper expertise into your project.

Additional Reading

[1] Control function measurement techniques can be found at <http://www.ridleyengineering.com/downloads/AP-200Notes.pdf>

[2] Read about the effect of filters on switching regulators: <http://www.switchingpowermagazine.com/downloads/Evolution of Power Electronics.pdf>

[3] For more reasons for convert instability, read: <http://www.switchingpowermagazine.com/downloads/9 Six Reasons Instability.pdf>